

# The Development of Failure to Learn Mathematics (FtLM): a discursive perspective

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Lecture at Montclair State University

Graduate Seminar

April 26, 2022



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# Failure to Learn Mathematics (FtLM) – huge amount of research, and yet still many gaps

Cognitive perspective: Explains FtLM as a problem in the development of a certain mental capacity

(Butterworth, 2005; Geary, 2011; Dowker, 2019)

- Mostly relates to FtLM as a state
- Main terms: Dyscalculia; Mathematical Learning Disability; Mathematical competence

Affective perspective: Explains FtLM (partially) in terms of “math anxiety” or “low math concept”, or “low motivation”

(Dowker et al., 2016; Ma, 1999; Ramirez et al., 2018)

- Relates to FtLM as a process (reciprocal relations between affect and achievements), but at a very macro-level
- Main terms: Math anxiety; math self-concept; motivation to learn mathematics; self-efficacy in mathematics

Social perspective: Explains FtLM in terms of marginalization, lack of opportunities to learn, and social narratives inhibiting engagement in learning

(Ben-Yehuda et al., 2005; Martin, 2000; Tan et al., 2019)

- Relates to FtLM mostly as a state, sometimes as process, rarely relates to the mathematical content
- Main terms: race; gender; home-environment; SES

Cognitive explanation: FtLM happens because of inherent/developmental deficits

- But then: why is math anxiety so strongly correlated with achievements (and the relation is bi-directional)?
- Why are there more MLD in low SES?
- Why is math achievement so strongly correlated with SES? With Gender (in certain countries)?

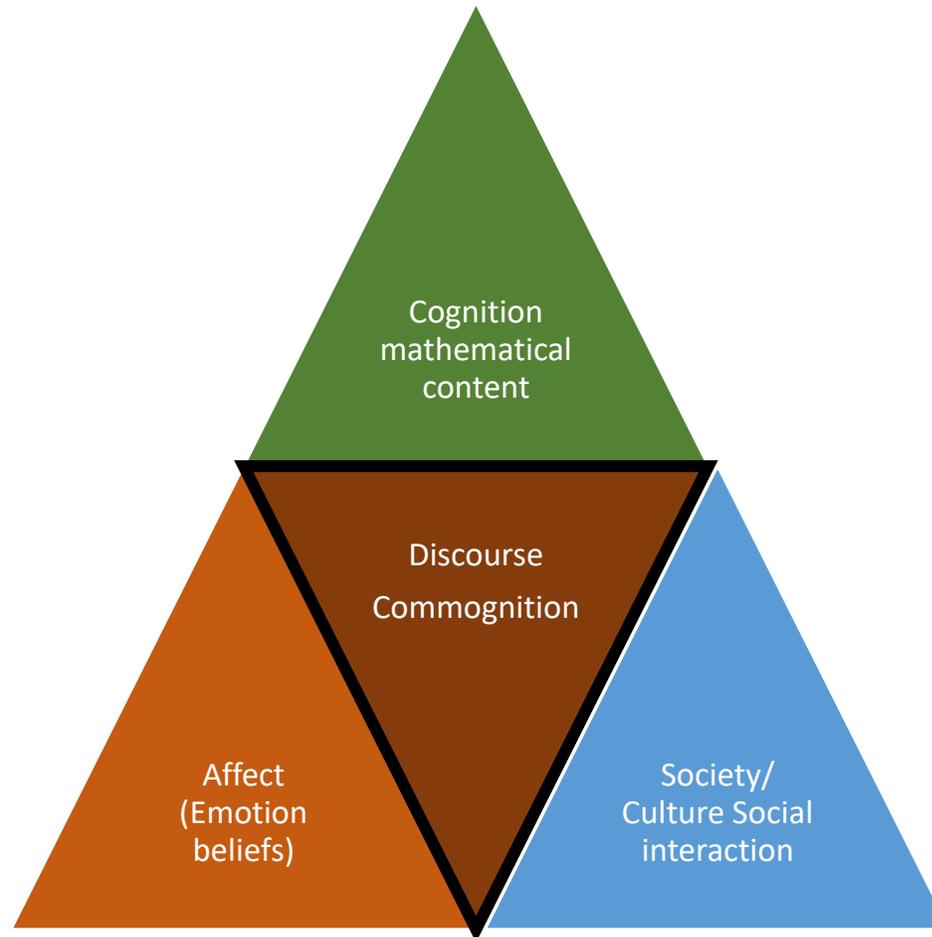
Affective explanation: FtLM happens b/c students develop apprehension towards math

- But then – why do certain cultures have high incidents of anxiety and high achievements?
- Where does the anxiety come from? How does it develop?
- How does the anxiety affect learning (not just test taking)?

Social explanation: FtLM happens b/c students of certain identities are not given sufficient OTLs

- But then – why do certain students with similar identities fail and other succeed?
- How does the identity label impact OTLs?

# There is need for a holistic perspective



# Commognition (Sfard, 2008) in a nutshell

## Major underlying assumptions

- Thinking is an intra-personal form of communication
- Mathematics is a discourse
  - Defined by certain keywords, visual mediators, narratives and routines
- Learning is becoming a participant in a community

If we want to study learning, let us study students' discourse

## The theory

- Learners start by participating ritually – imitating others
- Gradually, their participation becomes more explorative, as they master the routines of the discourse

# Identity in communicational terms

“A collection of stories, told about a person, that are reifying, significant and endorsable”

(Sfard & Prusak, 2005)

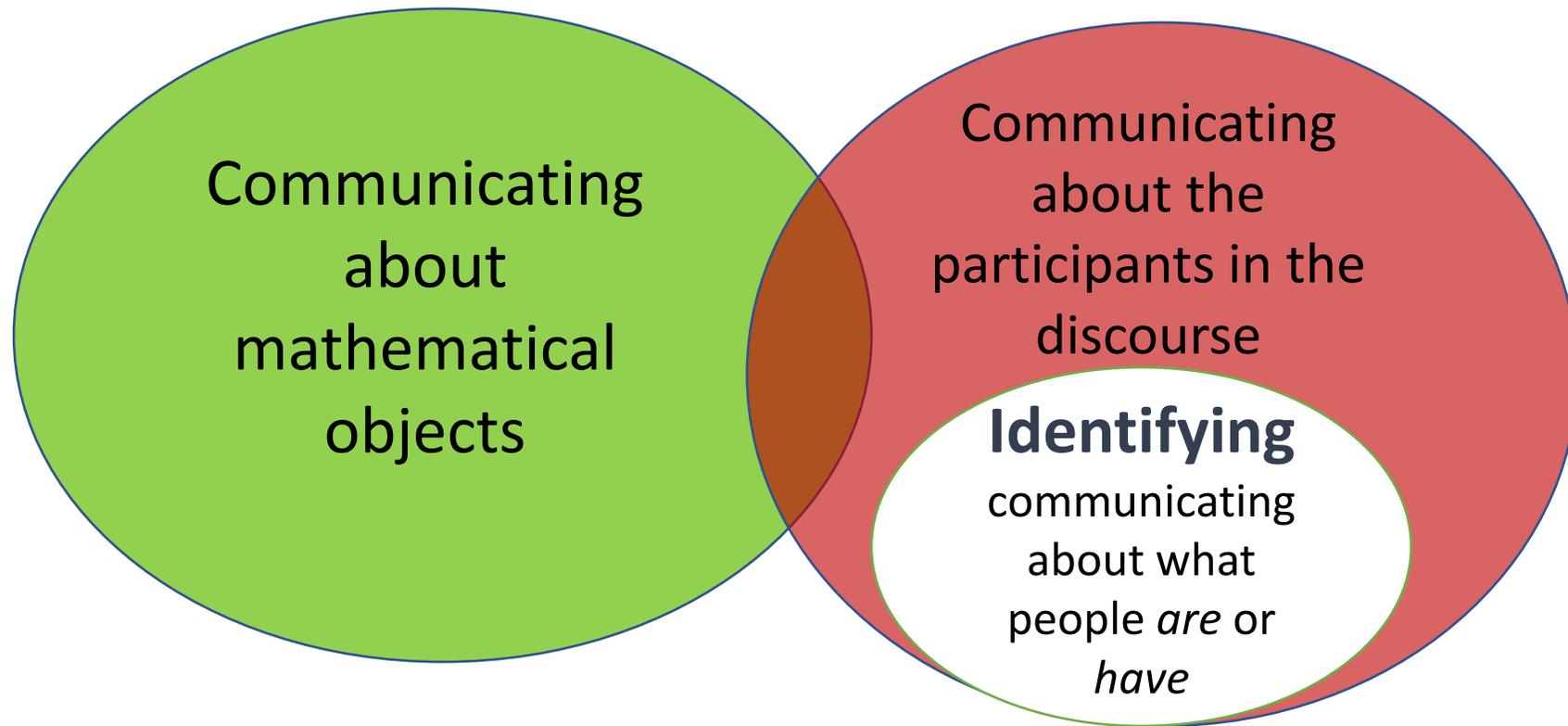
Types of identity stories:

- 1<sup>st</sup> Person, 2<sup>nd</sup> P, 3<sup>rd</sup> P stories
- Current identity: stories about the current situation
- Designated identity: stories about how things should be

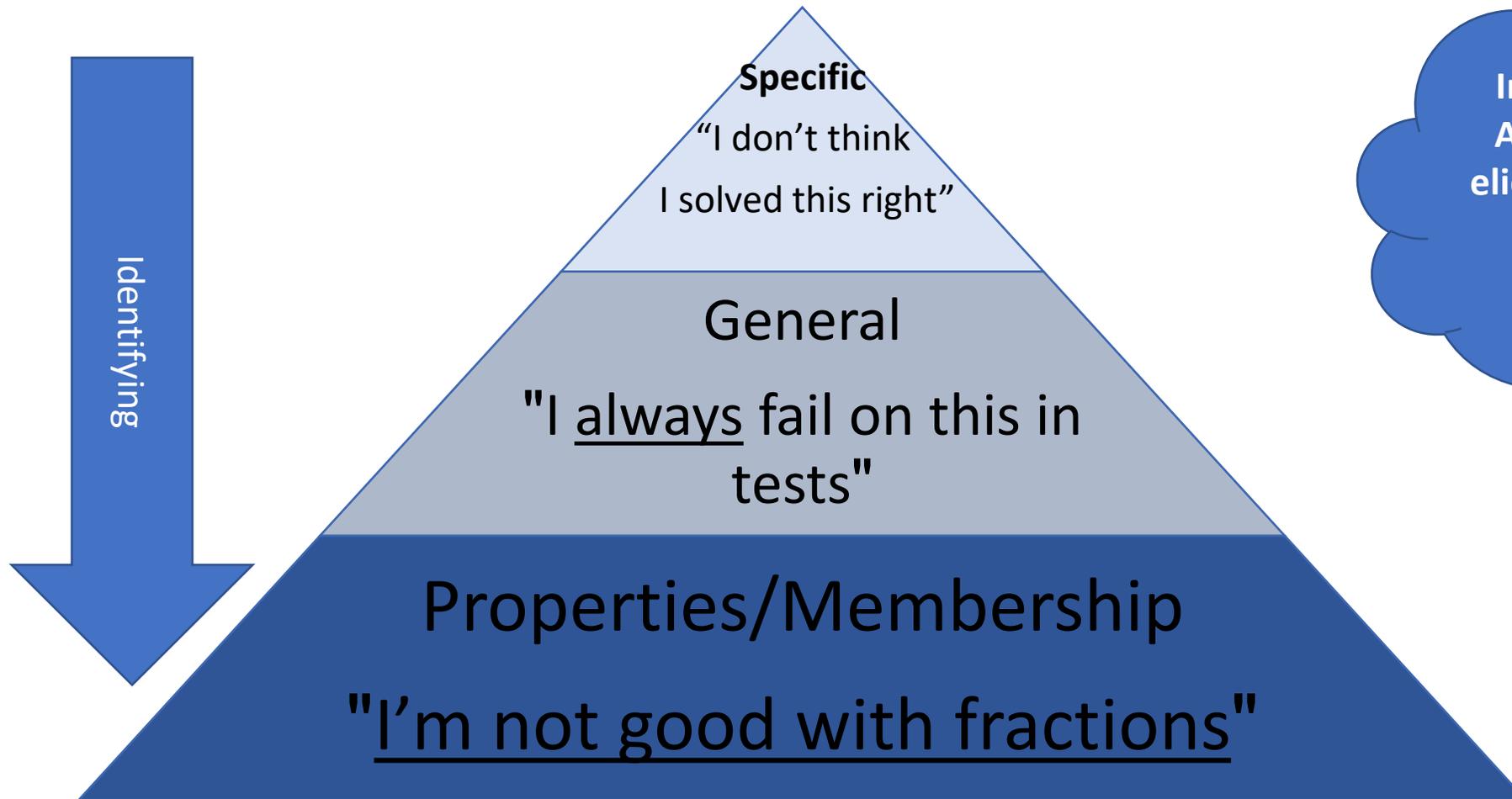
# From Identity to Identifying (product to process)

## Mathematizing

## Subjectifying



# Levels of subjectifying

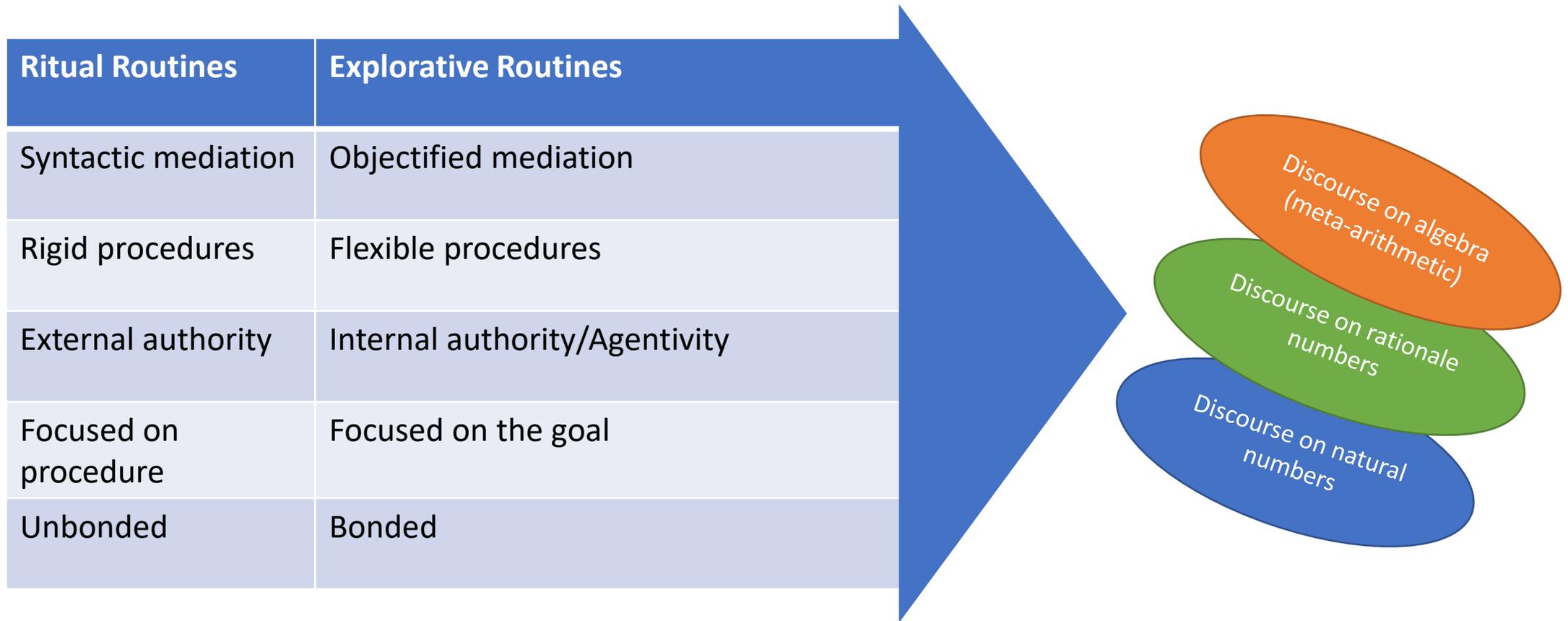


**Implicit identifying:**  
Actions intended to elicit in others identity narratives about oneself

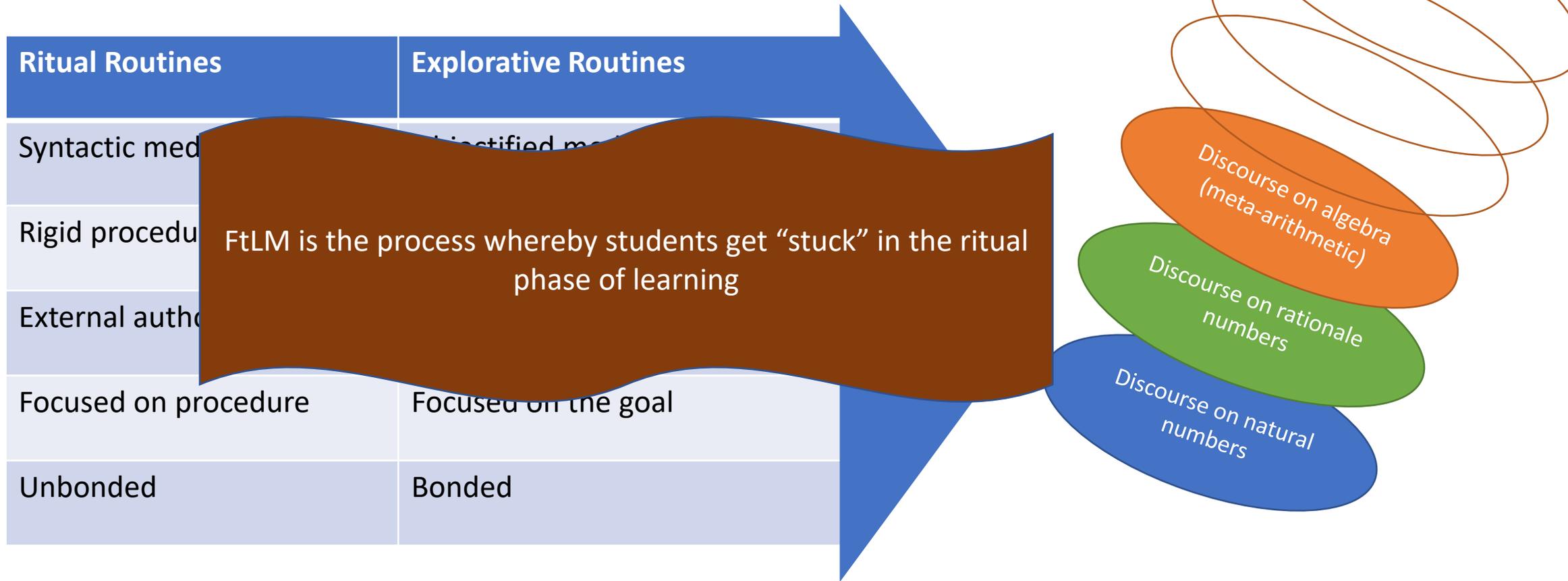
# Failure to Learn Mathematics - A discursive definition

A consistent impasse to developing an identity of inclusion in a certain mathematical community/discourse

# Learning as progressing from rituals to explorations



# Learning as progressing from rituals to explorations



# The story of Idit

- 7<sup>th</sup> Grader (Age: 13)
- Moderate-high achievements
- Identifies herself as generally excelling in math
- Her only problem is that she's "not good with fractions"
- Participates in my out-of-school course, succeeds at final year exams, gets tracked to the highest track
- End of 9<sup>th</sup> grade: Idit fails math; gets tracked to the lowest (basic) high-school math track
- Reports severe "math anxiety"



# Idit's 1<sup>st</sup> math interview – 7<sup>th</sup> grade

Write  $>$ ,  $<$  or  $=$        $1/5$    $1/7$

**Idit:** So Um.. (looks at interviewer, smiles) here I'm completely unsure, I told you I'm not good with fractions. (Looks back at the sheet), so I think... that this is bigger (points to  $1/5$ ).. [I'm] not sure.

**Interv:** Why do you think so?

**Idit:** 'cause, if I'm not mistaken, in elementary school we were told that the smaller the number, so it's bigger. That's what we were told (shrugs shoulders, slightly smiles).

# Mathematizing

*The task: write  $>$ ,  $<$  or  $=$ :  $1/5$  []  $1/7$*

**Idit:** So Um.. (looks at interviewer, smiles) here I'm completely unsure, I told you I'm not good with fractions. (Looks back at the sheet), so I think... that **this is bigger (points to  $1/5$ )**.. [I'm] not sure.

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# Subjectifying

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# Identifying

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# Analysis of ritual/explorative participation

*The task: write >, < or = 1/5 [] 1/7*

Reified identity narratives of failure (specific to fraction)

**Idit:** So Um.. (looks at interviewer, smiles) here I'm completely unsure, I **told you I'm not good with fractions.** (Looks back at the sheet), so I think... that this is bigger (points to 1/5).. **[I'm] not sure.**

Lack of agency

Reliance on external authority

**Interv:** Why do you think so?

**Idit:** 'cause, if I'm not mistaken, **in elementary school we were told that the smaller the number, so it's bigger.** That's what we were told (shrugs shoulders, slightly smiles).

Syntactic mediator  
1 / 5 / 7 treated as separate signs

# Idit's discourse on natural numbers

Idit: (Reads the question) Um.. Four shirts cost 200 \$, six pairs of pants cost 300\$. Rina bought two shirts and two pairs of pants. How much did she pay? So she bought two shirts which is 100 \$ (looks down)

Interv: Umhmm

Idit: And two pants which is... also 100 \$

Interv: OK

Idit: (Writes) so that gives 200\$

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Interv: Umhmm

Agency, internal authority

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Agency, internal authority

Interv: OK

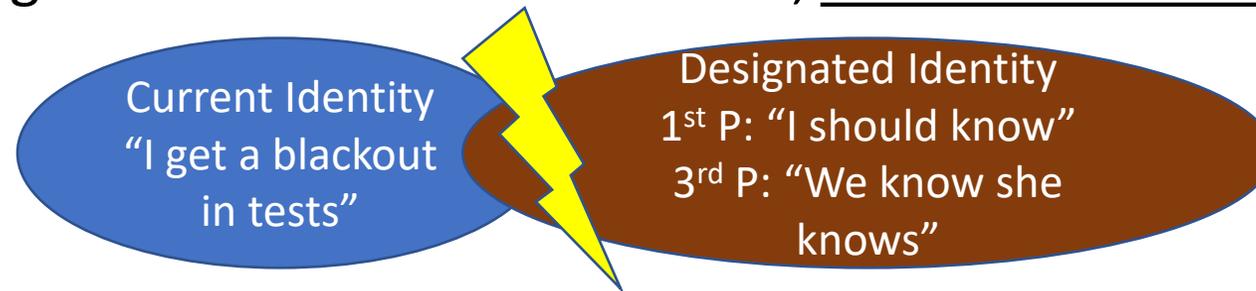
Idit: (Writes) **so that gives 200\$**

Objectified mediator

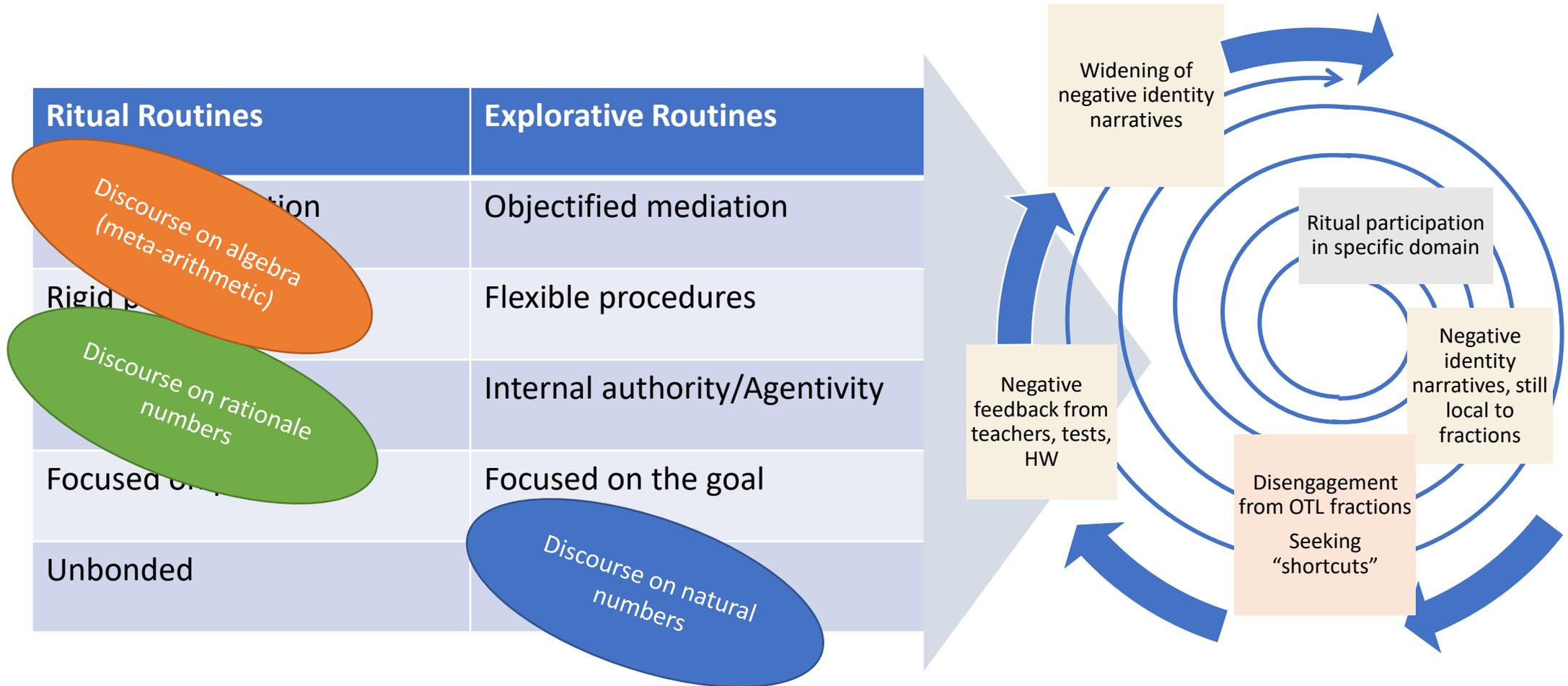
# Idit's identity narratives at 9<sup>th</sup> grade

Idit: “The instant I get to a test it like, gives me a blackout, that I forget everything. Now, before the test, I know all the material, and- and I go over everything, I solve all the exercises in the book and I know everything by heart. And- and when I get to the test I have this, like suddenly this anxiety that makes me forget all the material.”

Mother: “She has an expectation of herself too, (its) not just that she knows that we have an expectation (of her). But we don't come (.) or punish (her). She knows that we just care that her grades reflect what she knows, we know that she knows.”



# Idit's hypothesized widening failure



# Dana's hypothesized widening failure – probably started much earlier

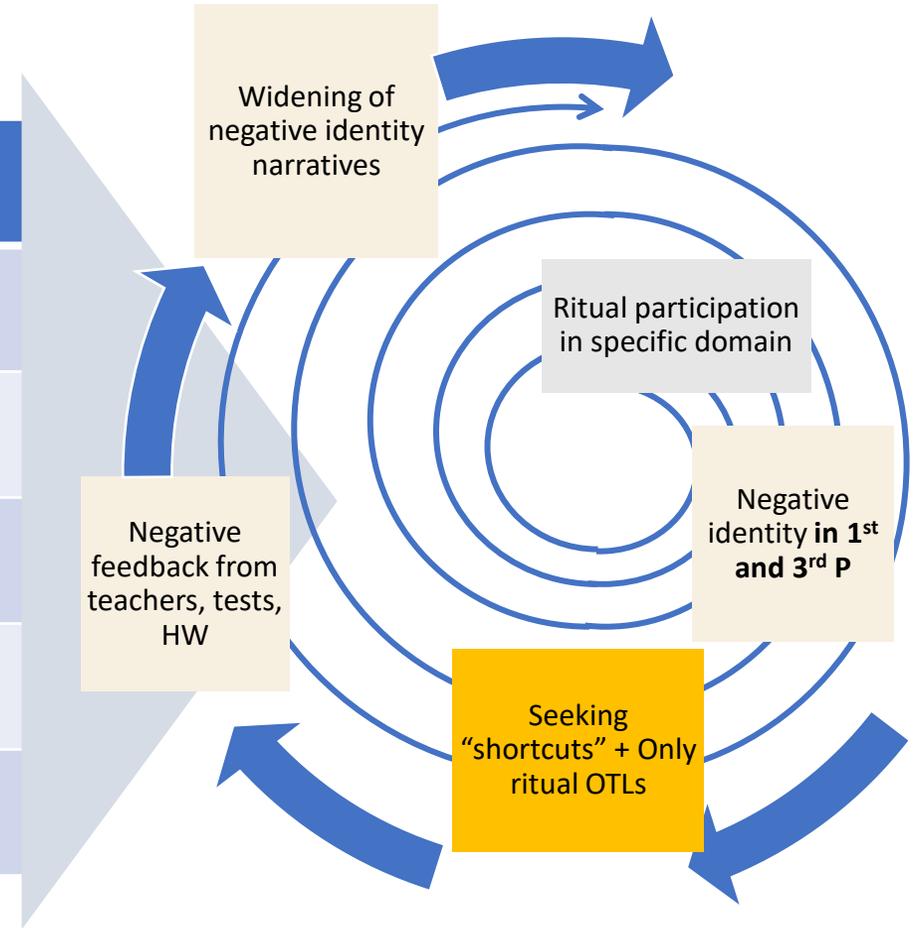
Ritual Routines	Explorative Routines
Artic mediation	Objectified mediation
Procedures	Flexible procedures
Internal authority/Agentivity	Internal authority/Agentivity
Focused on the goal	Focused on the goal
Bonded	Bonded

Discourse on algebra (meta-arithmetic)

Discourse on rationale numbers

Discourse on natural numbers

Discourse on numbers between 1-20



# Researching ways to assess and intervene with FtLM



Prof. Michal Tabach,  
Tel-Aviv University

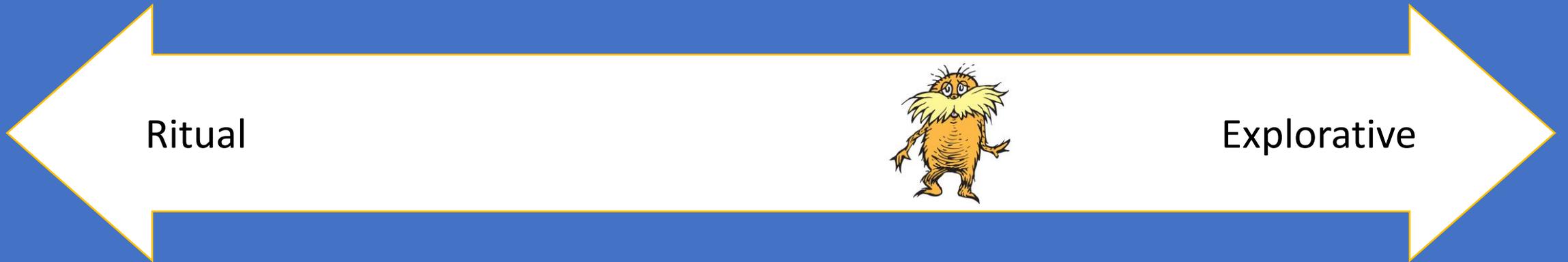
- The LATID project: **L**earning **A**lgebra through **T**echnology for developing mathematical **I**dentities
- The main problem:
  - If students arrive at 7<sup>th</sup> grade with a ritual arithmetic discourse how can they ever participate exploratively in the meta-arithmetic discourse = algebra?
- The solution:
  - By using Excel activities: Provide students with scaffolds to “bypass” the ritual routines in arithmetics, so they can participate exploratively in the discourse of algebra

# The LATID project: Learning Algebra through Technology for developing mathematical Identities

- Four year project (Current: Year 2/4)
- Stages:
  - Develop diagnostic tools to assess arithmetic and early algebraic discourses
    - RDP – aRithmetic Discourse Profile
    - EADP – Early Algebra Discourse Profile
  - Intervene with Excel based activities
  - Measure the effectiveness of B with the tools developed in A, as well as through analysis of learning processes.

Developing diagnostic tools

# The aRithmetic Discourse Profile (RDP)



RQ: Based on tasks taken from elementary school curriculum, how can students' arithmetic discourse be characterized on the continuum between ritual and explorative participation?

# Method

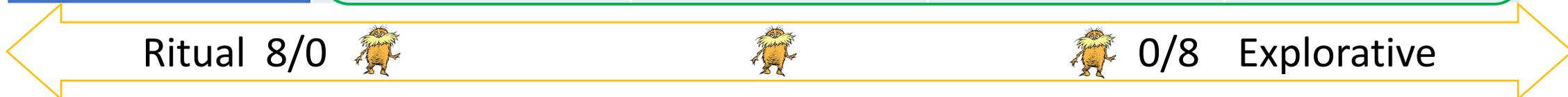
- 12 7<sup>th</sup> grade students (Heyd-Metzuyanim, 2011), interviewed on arithmetic tasks using a “think aloud” protocol
- 3 tasks chosen for the present analysis:  $96+7935$ ;  $25 \times 99$ ;  $\frac{2}{3} \times 9$
- Verbatim transcriptions
- 1<sup>st</sup> Step of analysis: dividing each routine into subroutines
- 2<sup>nd</sup> Step: determining ritual/explorative characteristics of the routine

# Criteria of analysis for ritual-explorative routines

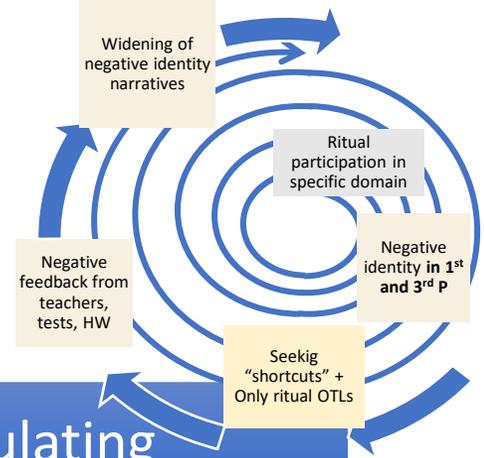
	Criterion	Analytical actions	Characteristics of an explorative routine	Characteristics of a ritual routine
1	Objectified /syntactic mediation.	Searching for evidence that the nouns signify numbers/quantities and not just the signifier of the number	In whole numbers, relating to the place value of the numeral. In fractions: relating to different realizations of the fraction as the same, including fraction as operator, part of whole, part of quantity, etc.	In whole numbers: relating to operations as signaling procedures on digits rather than on the whole number. In fractions, relating separately to the numerator and denominator.
2	Flexibility	Look for multiple procedures that are associated with the same task.	More than one procedure is associated with the main task OR a non-standard procedure is applied to the task.	Relying on only one procedure while showing rigidity and reluctance to use any other procedure.
3	Agency/External authority	Look for subjectifying discourse; examine verbs/pronouns and non-verbal signals that indicate the confidence.	Mathematizing with high confidence (no hesitations, question marks, no looking for approval).  Spontaneously articulating mathematical narratives	Talking with question marks; Verbally or Non-verbally seeking approval from the interviewer; Relating to external authority for justification (e.g. "that's what I learned in school")
4	Focus on goal or on procedure	Look for verbs indicating doing (e.g. "I add") vs. being verbs indicating the result ("it is...");	Talking about the result, checking it, or explaining it spontaneously	Talking about the actions of the procedure. Ending the procedure without relating to the reasonableness of the result.
5	Bondedness	Examine the procedure; sub-procedures and the bonds between them.	Each sub-procedure feeds the next sub-procedure. The narrative of the result of sub-procedure N serves as the input of sub-procedure N+1.	There is a disconnect between a certain sub-procedure and its following one OR sub-procedures using different realizations are not treated as the same.

# Findings – Ritual-exploration ratios and relative placement of the 12 students

Student	Achievement group	96+7935 Ratio Ritual/Exploration	25×99 Ratio Ritual/Exploration	$\frac{2}{3} \times 9$ Ratio Ritual/Exploration
Dana	Low	6/4	8/2	7/3
Hili	Low	7/2	Not attempted	Not attempted
Hila	Low	7/3	7/1	Not attempted
Naor	Low	2/6	8/3	4/5
Edna	Middle-high	8/3	6/7	Not attempted
Idit	Middle-high	0/7	6/5	7/4
Dan	Middle-high	1/7	7/6	2/6
Ziv	Middle-high	3/6	3/8	2/6
Ram	Excelling	0/6	0/8	1/7
Gabby	Excelling	0/7	0/8	1/7
Yoram	Excelling	0/7	0/8	0/7
Amir	Excelling	0/7	1/8	0/8



# FtLM - Summary



The commognitive explanation of FtLM is theoretically robust, but accumulating empirical evidence is difficult

- Commognitive analysis is very work-intensive
- Demands longitudinal analysis over long periods of time, with very high engagement from participants

Another challenge lies in the *incommensurability* of commognition with other “cognitivist” theories

- Without a “common language”, building on previous studies that have used cognitivist theories is extremely difficult

**Work in progress:** a critical review with the aim of understanding how the highly fragmented literature on FtLM can be built upon and viewed through a discursive lens

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