Extending Mathematical Practices to Online Teaching

The NCTM Process Standards provide a window to analyze two lesson excerpts.

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During a lesson in which students are studying representations that model a \$15 weekly expenditure, one student raises a question about a geometric representation: Thinking of the expenditure as a slope of –15/1, he asks, "Where did you get the –15/1?" The professor hesitates; she anticipates using verbal explanations and fraction representations. She cannot reach for a marker or chalk. She asks herself, "What are my options, and what are their pros and cons?"

Why is this professor, who is an experienced teacher, struggling to respond to a relatively familiar student question? Part of the answer has to do with the fact that she (and her students) are newcomers to synchronous online teaching.

We define *synchronous online teaching* as a situation in which a teacher and students sign on to a conferencing program—from different locations but at the same time—to participate in a lesson. Such programs can be useful when face-to-face meetings are not feasible but teachers still want to provide support, enrichment, or lessons to students. Such programs can also modify how teachers provide support or enrichment and how teachers conduct lessons.

In this article, we describe a specific case of online teaching. Our goal is to familiarize teachers who are interested in synchronous online instruction with some of the modifications and possibilities that can arise in this new environment. We examine and discuss these issues in a course that was taught partly with the conferencing program ElluminateLive® (now rebranded as Blackboard Collaborate™). We begin by discussing two general issues that teachers face as they consider using online platforms.

TWO CONSIDERATIONS FOR ONLINE TEACHING

Every teacher who is new to online teaching will need to become familiar with the user interface. For example, Elluminate's interface includes a participant window for participant names; a chat window where students and moderator, usually the teacher, can type; and a microphone feature for students and teacher to use when talking (see **fig. 1**). A whiteboard is used for writing and drawing or

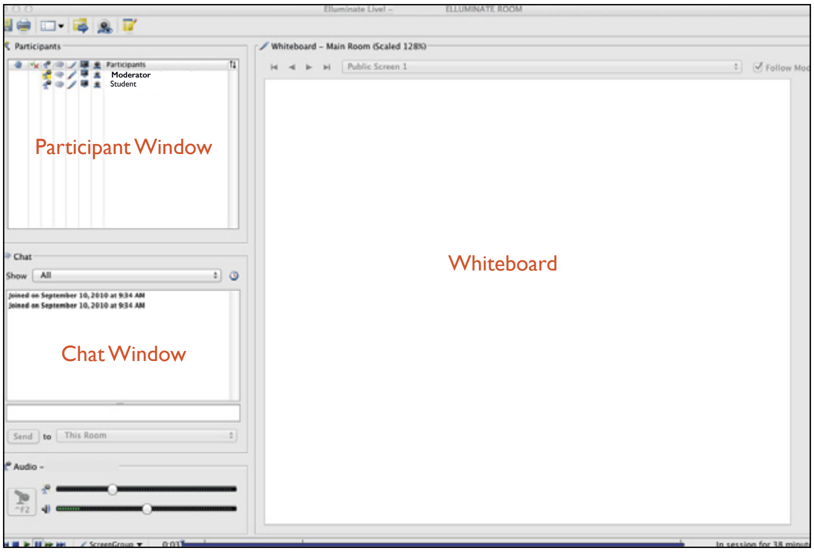

Fig. 1 A typical Elluminate window is annotated with red text

for displaying computer applications through a feature called application sharing. Readers who recognize these features in other conferencing programs can apply this article's issues to their programs despite the particulars of their own environments.

A second issue concerns the relationship between the new teaching environment and a teacher's beliefs. The relative newness of online technologies can lead teachers to raise questions about their pedagogies and about the technologies themselves (De Gagne and Walters 2009; Mishra and Koehler 2006). Mathematics teachers may be skeptical about whether these technologies can support recommendations advocated by the math education community (NCTM 2000, 2014). Such questions and skepticism can prevent teachers from experimenting with these new technologies and with their role in teaching.

SYNCHRONOUS ONLINE TEACHING AND THE PROCESS STANDARDS

To explore these issues, we connect one professor's work with Elluminate to the NCTM Process Standards (2000)—problem solving, reasoning and proof, connections, communication, and representation—which directly influenced the professor's beliefs and her design of the online course.

The course, a 100-level mathematics course taught at a state university, is typically taken by undergraduates who are mathematically anxious and struggle to place into "college-level" mathematics. The professor designed the course, created its materials, and posted them to the university's learning management system (LMS) (Fernández 2014). The "Standards-like" beliefs underlying her process included "developing problem solving autonomy and collaboration, encouraging critical thinking, exposing students to the full spectrum of mathematical thinking (from straightforward calculation to open-ended investigation) and giving students the mathematical tools and disposition to help them interpret and enrich their lives." (Fernández 2014, p. 2). Upholding these beliefs in a synchronous online setting presented her with a new and challenging learning experience.

The class focused on algebraic reasoning and on using and interpreting multiple representations and spreadsheets to study finance models. The materials created opportunities for individualized work, such as viewing videos, reading, and problem solving. For students who wanted additional (and more collaborative) help, the professor arranged a weekly meeting time on Elluminate. Students could attend these sessions, after completing the week's work, and bring questions to guide the meetings. Thus, the students' questions, rather than a professor's plan, drove the lessons. During these meetings, students opted to communicate using the chat window, while the professor used the microphone. These sessions, which were recorded and posted onto the LMS, are the focus of this article. All student names used in this article are pseudonyms.

THE FIRST VIGNETTE

If a class is held in a synchronous online environment, how does the process of doing and teaching mathematics change? Can the teacher and the students uphold recommendations made by the mathematics education community? Consider the opening vignette in which the professor becomes conflicted over how to address the question her student, Mark, poses, which is "Where did you get –15/1???" (see **fig. 2**). The reading for the week discussed linear earning and spending habits by using verbal, tabular, algebraic, and geometric representations. Mark's question stems from the problem of determining when a boy with $150, who spends $15 per week, will run out of money. The professor brings up the reading in the whiteboard window, and Mark directs her to the section where this problem's features are connected to the geometric techniques of plotting (0, 150), "rising" –15 dollars, and "running" 1 week, until the solution is identified.

Choosing a Format to Address the Student's Question

Mark communicates an "object of . . . discussion" (NCTM 2000, p. 60) by typing a question into the chat window about the origin of "–15/1." Instead of listening, the professor and students read his inquiry. Reading the typed "–15/1" in this graphing context prompts the professor to help students see the connection between "–15/1" and the section's

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$$\frac{rise}{run}$$

(see **fig. 2**). For the professor, this raises the issue of creating the displayed vertical alignment

$$\frac{-15}{1}$$

within this new environment so that *rise* and *run* align, respectively, with –15 and 1. This opens up possibilities for relating the Representation Standard to the "process" (NCTM 2000, p. 67) of re-creating a representation.

The professor considers the benefits and shortcomings of Elluminate's options for creating her representations: The whiteboard closely simulates a face-to-face classroom, but the writing tool is limited and sloppy; Microsoft® Word presents text clearly but does not permit an alignment of numerator over denominator; and Equation Editor permits a proper alignment but is clumsy to use. She decides on Equation Editor and re-creates

$$\frac{-15}{1}$$

with accompanying explanation and questions (see **fig. 3**). This resolves Mark's confusion.

Next, Mark analyzes this situation in "mathematical terms" (NCTM 2000, p. 53) and types, "so run will always be 1?" This question introduces an opportunity to engage students in the nuances of reasoning and proof, since Mark is making a generalization on the basis of one observation. The professor considers introducing rises such as –30 or –45. Because it is their first online meeting, she also considers how these numbers, which are different from the problem's numbers, might distract students away from the logic underlying counterexamples. She opts to retain the investigation's numbers and weekly increments and introduces a counterexample that challenges students to find the run when the rise is 15 so that

$$-15 = \frac{15}{?}.$$

Students resolve that the run will be –1.

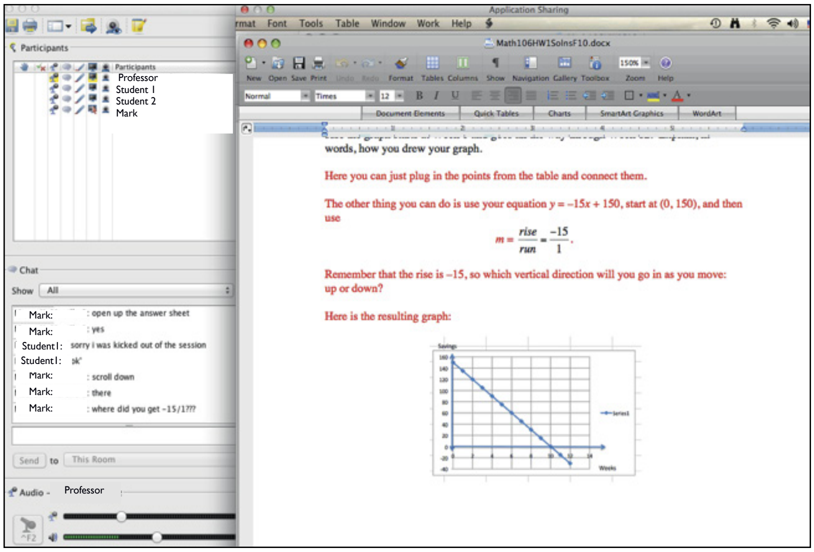

Fig. 2 Mark's question is displayed in the chat window. The reading section is displayed in the whiteboard.

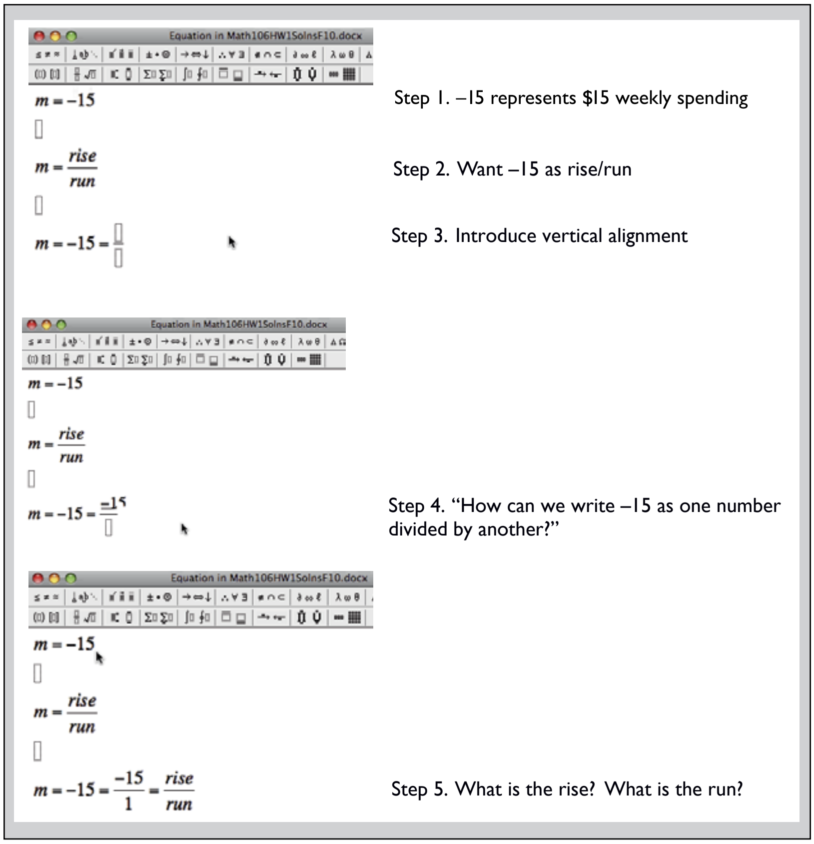

Fig. 3 These screenshots show the professor's recreations of necessary representations in Equation Editor using Application Sharing.

The Process Standards in the Opening Vignette

This vignette lasted only 3 minutes 35 seconds. Even though it was brief, the exchange offers insights into changes that can arise in an online setting, as well as how to be mindful of the Process Standards in that setting.

In synchronous online settings, teachers and students can expect to communicate by typing into chat windows, reading common screens (from different locations), speaking into microphones,

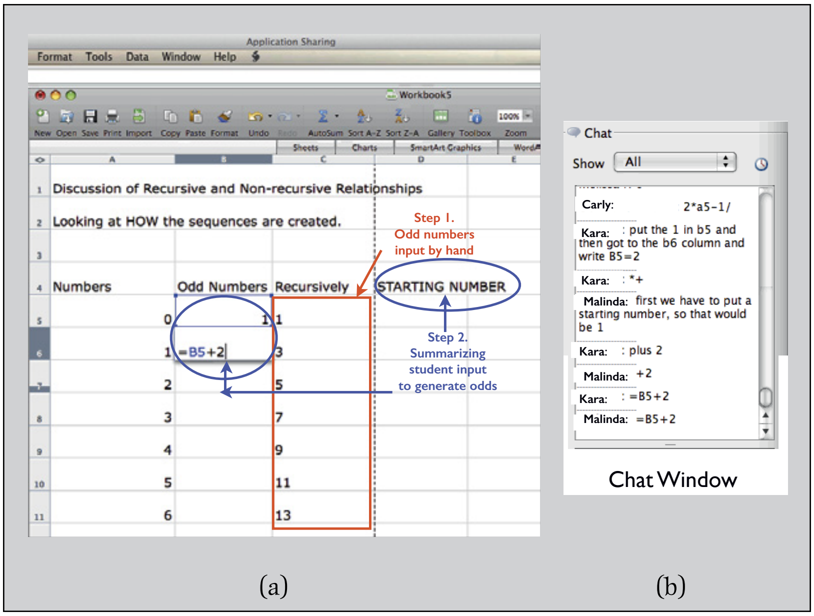

Fig. 4 This screenshot, annotated with red and blue text, shows students' attempts to generate odd numbers recursively (a), as well as the corresponding dialogue in the Chat Window (b).

and calling up computer applications. In the opening vignette, reading a student's written question prompted the professor to focus on an aspect of representation that may have been taken for granted in the verbal exchange of a face-to-face setting. In attempting to connect the numerical and geometric representations, the professor deliberated about the pros and cons of developing the representations with different technological options. Teachers need preparation for, and exposure to, such negotiations so that they can make a chosen technology work for the online situations they find themselves in.

Mark's second question opened up a pathway to asking, "Is this true always?" (NCTM 2000, p. 57). The professor used this opportunity to create a situation to reason about a counterexample. While Mark's first question raised technological concerns related to the online setting, the professor's deliberations at this juncture are based on her knowledge of the students, the content, and pedagogy. That is, the reasoning resembles that used in a face-to-face setting. Teachers can expect to continue making such choices in online settings, generating other possibilities for reasoning.

Finally, this episode introduces an opportunity to consider the different places where learning can happen. The current course supports student learning through readings, videos, or optional synchronous conferencing sessions driven by students' questions. This combination of opportunities suggests that the online sessions may not reflect the entire treatment of material learned. For example, interpretations involving rate of change and mathematical modeling are discussed in the readings and videos, but they are not discussed in this online episode, since no student raised them. As technologies continue to expand learning experiences (NCTM 2000, pp. 70–71), teachers must consider how the technologies will come together in their own online courses to generate different venues for student learning.

Teachers can expect new challenges and opportunities as they process students' written communication within synchronous online settings.

THE SECOND VIGNETTE

This section's episode has two distinguishing features: (1) It focuses on student discourse in the chat window and (2) most of the mathematical reasoning took place on a spreadsheet. The content under study was recursive and nonrecursive relationships. As an exercise, students were asked to design a spreadsheet that generated even numbers recursively and nonrecursively. During that week's meeting, multiple students asked how to apply the same exercise to odd numbers, articulating a "curiosity" (NCTM 2000, p. 53) about a situation related to one they had studied.

Student Discourse in a Chat Window

The professor manually input odd numbers into a spreadsheet column and asked students to generate the sequence recursively (see **fig. 4a**, step 1 annotation). The participant window signaled to the professor that students were typing simultaneously, so she awaited their responses. In the chat window, the professor read Carly's nonrecursive formula, Kara's attempt at a recursive formula, which she self-corrected, and Malinda's more general statement about how to initiate recursive sequences (see **fig. 4b**). The professor, recognizing an opportunity for reordering these responses, first articulated Malinda's observation about a starting number, then input this number and Kara's recursive formula (see **fig. 4a**, step 2 annotation).

Moving to a nonrecursive formulation, the professor cleared column C (see **fig. 4a**) and worked with Carly's formula "2*a5 – 1 " (see **fig. 4b**, chat window). The professor and the students discussed the error this generated in cell C5 and constructed a new formula, "(2*A5) + 1." At this point, Carly reassessed her original formula and typed, "I think my formula should have been 2*A6 – 1."

The professor cleared columns B and C and inserted "=2*A6–1" into cell B5 (see **fig. 5a**). She dragged the formula down, asserting its validity and originality, when Kara typed, "but six is not right" in the chat window (see **fig. 5b**). Perplexed, the professor considered Kara's observation and realized the sixth entry is not producing 13. The professor questioned, "Hold on. Why didn't that work? Oh! You know why? Tell me why that didn't work!" Students typed different versions of the same observation (see **fig. 5b**): Carly's formula needed an input that is diagonal to the formula entry. The professor reinforced students' reasoning by highlighting relevant spreadsheet cells (see **fig. 5c**) and dragging the formula in A11 to A12 to produce the necessary input.

At the conclusion of this episode, Kara typed her disbelief that this unconventional formula constituted a viable response: "I did not know we could do that!" By supporting Carly's approach, the professor helped Kara learn the value of exploring alternatives to the usual formulas generated. The episode additionally illustrates students communicating by "carefully 'listening' to, and thinking about, the claims made by others"

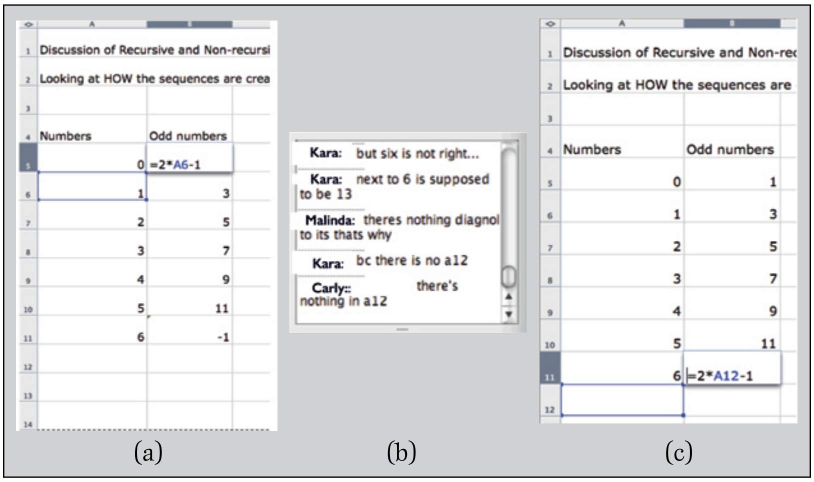

Fig. 5 These screenshots show the development of Carly's second formula.

(NCTM 2000, p. 63) through the "common referent" of the spreadsheet (NCTM 2000, p. 60). By outlining the cells in **figure 5c**, the professor connected algebraic and technological considerations to students' reasoning.

The Process Standards in the Second Episode

Teachers can expect new challenges and opportunities as they process students' written communication within synchronous online settings. The simultaneous display of multiple ideas, the order in which ideas appear, and the articulation of unanticipated student ideas can affect the teacher's role in synchronous online platforms.

As students are encouraged to approach a problem from different points of view (NCTM 2000, p. 62), online chat windows can create written records of these viewpoints. A chat window's dynamic responsiveness allows for immediate, uninterrupted, and independent student thought, as well as communication of, and responsiveness to, these viewpoints. In the episode above, this feature enabled Kara to self-correct her formula multiple times. It enabled Carly to "adjust [her] strategy" (NCTM 2000, p. 54) and communicate a "different approach" (NCTM 2000, p. 54) for generating her nonrecursive "diagonal" formula. Written records can also influence the order in which students' ideas are used, as demonstrated by the professor's reordering of Kara and Malinda's solutions so that both could contribute meaningfully to the lesson. With these new interactions, teachers will need to learn to negotiate

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multiple written communications so that students' viewpoints can contribute to a lesson's development.

Students' unanticipated responses can introduce additional challenges to teachers in online settings. When Carly's revised "2*A6 – 1" produced a "–1" instead of "13," the professor failed to see what was going wrong. Teachers certainly can fail to see ideas in face-to-face settings as well, but mechanisms such as gesturing and speaking can facilitate communication when such obstacles arise. In this online episode, the professor first articulated her confusion and then resolved it for herself. When she invited students to resolve the problem for themselves, students were able to use spreadsheet cell references to direct her attention to the missing input. By taking advantage of the features available to them, the professor and the students supported one another to make Carly's formula work and, in the process, demonstrated that students "share responsibility with the teacher for the learning that occurs in the lesson" (NCTM 2000, p. 61).

A PLATFORM FOR LEARNING

In this article, we described some of the modifications teachers can expect if they use synchronous online platforms, as well as the possibilities for upholding ideas inherent in the NCTM Process Standards. We encourage the reader to continue searching for examples of student-driven lessons and of students revising their own or their peers' work, introducing problems, making discoveries, teaching their teacher, publicly reasoning, and creating valid representations after multiple tries.

We conclude with an observation: Current calls about teacher training in online technologies recognize that "knowing how to use technology is not the same as knowing how to teach with it" (Mishra and Koehler 2006, p. 1033). Issues specific to content, teacher knowledge, and student thinking in a discipline must be integrated with technology to support teachers' efforts to use these formats. Our episodes additionally demonstrate the importance of teacher beliefs in making decisions, solving problems, and responding to students' thinking in online settings. We invite teachers and teacher educators to consider how to challenge or uphold their own beliefs, in conjunction with the factors cited above in the literature, so that students can have meaningful learning experiences with mathematics in online settings.

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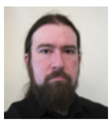
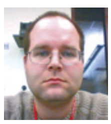

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